Gauging a quantum heat bath with dissipative Landau-Zener transitions

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I. Abstract

We calculate the *exact* Landau-Zener (LZ) transition probabilities for a qubit with arbitrary linear coupling to a bath at $T = 0$ (K). The bath causes time-dependent relaxation of the qubit; dephasing has little or no influence. Applications include circuit QED [1,2] and adiabatic quantum computation. A qubit undergoing LZ transitions is a robust "bath detector" [3].

II. Introduction: Landau-Zener transitions

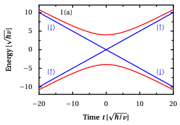
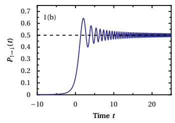


Fig. 1a: Standard LZ problem (1932):

$$H_{LZ}(t) = \frac{vt}{2}\boldsymbol{\sigma}_z + \frac{\Delta}{2}\boldsymbol{\sigma}_x.$$

Exactly solvable dynamics (**Fig. 1b**), rare in driven quantum mechanics. Transition probability

$$P_{\uparrow\to\downarrow}(\infty) = 1 - e^{-\pi\Delta^2/2\hbar v}.$$



How does $P_{\uparrow\to\downarrow}(\infty)$ change when the qubit couples to a bath at $T = 0$ (K)?

$$H(t) = H_{LZ}(t) + \sum_{j=1}^{N} \hbar\Omega_j b_j^\dagger b_j + (\cos\theta\,\boldsymbol{\sigma}_z + \sin\theta\,\boldsymbol{\sigma}_x)\sum_{j=1}^{N}\frac{\gamma_j}{2}(b_j + b_j^\dagger).$$

Expect effects of bath-induced dephasing $\propto \cos\theta$ and relaxation $\propto \sin\theta$.

III. Method and "No-go-up theorem" [1,3]

Method: The interaction $V = \boldsymbol{\sigma}_x[\Delta/2 + \sin\theta\sum_{j=1}^{N}(\gamma_j/2)(b_j + b_j^\dagger)]$ enables bit flips. A polaron transformation diagonalizes $H_0(t) = H(t) - V$ with eigenstates $\{|\uparrow,\mathbf{n}_\uparrow\rangle, |\downarrow,\mathbf{n}_\downarrow\rangle\}$. Starting in initial ground state $|\uparrow,\mathbf{0}_\uparrow\rangle$, what is the survival probability $P_{\uparrow\to\uparrow}(\infty)$? Calculate the interaction-picture evolution operator $\tilde{U} = \overleftarrow{T}\exp[-(i/\hbar)\int_{-\infty}^{\infty}d\tau\,\tilde{H}(\tau)]$. Hamiltonian $\tilde{H}_{\mathbf{m},\mathbf{n}}(t)$ involves the time-independent matrices $W_{\mathbf{mn}}^{\pm}$ as given in Ref. [3],

$$\tilde{H}_{\mathbf{m},\mathbf{n}}(t) = e^{i(\mathbf{m}-\mathbf{n})\cdot\boldsymbol{\Omega}t}\{W_{\mathbf{mn}}^{+}e^{ivt^2/2\hbar}|\uparrow,\mathbf{m}_\uparrow\rangle\langle\downarrow,\mathbf{n}_-| + W_{\mathbf{mn}}^{-}e^{-ivt^2/2\hbar}|\downarrow,\mathbf{m}_-\rangle\langle\uparrow,\mathbf{n}_+|\}.$$

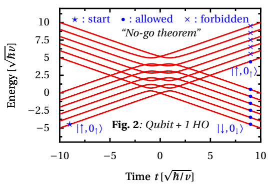
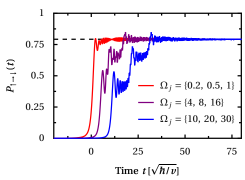


Fig. 2: *Qubit + 1 HO*

1. Selection rule ($\forall\theta$): Only $|\uparrow,\mathbf{0}_\uparrow\rangle \to |\downarrow,\mathbf{n}_\downarrow^{(1)}\rangle \to |\uparrow,\mathbf{0}_\uparrow\rangle \to |\downarrow,\mathbf{n}_\downarrow^{(3)}\rangle \to \ldots$ contributes to $P_{\uparrow\to\uparrow}$.
2. Qubit-bath entanglement [1]: Final state $|\psi(\infty)\rangle = \sqrt{P_{\uparrow\to\uparrow}}|\uparrow,\mathbf{0}_\uparrow\rangle + \sqrt{1-P_{\uparrow\to\uparrow}}\sum_{\mathbf{n}}c_{\mathbf{n}}|\downarrow,\mathbf{n}_\downarrow\rangle$.

IV. Transverse coupling ($\theta = \pi/2$) $\Rightarrow$ Relaxation [2]

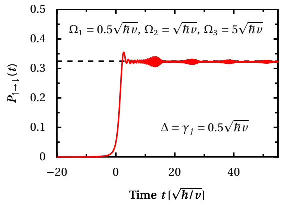
Spectral density $J(\omega) \equiv \sum_{j=1}^{N}(2\gamma_j/\hbar)^2\delta(\omega - \Omega_j)$. Exact LZ transition probability depends on integrated spectral density $S = \frac{\hbar^2}{4\pi}\int_0^\infty d\omega\,J(\omega)$:



$$P_{\uparrow\to\downarrow}(\infty) = 1 - e^{-\pi W^2/2\hbar v}, \text{ with } W^2(\Delta) = \Delta^2 + S.$$

Figure 3: LZ dynamics for a qubit with $\Delta = 0$, transversely coupled to three oscillators. Dashed line: analytical result for $P_{\uparrow\to\downarrow}(\infty)$. **All transitions are induced by the oscillators**.

V. Diagonal coupling ($\theta = 0$) $\Rightarrow$ Dephasing [3]



$P_{\uparrow\to\downarrow}(\infty) = 1 - e^{-\pi\Delta^2/2\hbar v}$: As for an *isolated* qubit!

Figure 4: LZ dynamics for a qubit diagonally coupled to three oscillators. Dashed line: *standard* LZ transition probability. $\Rightarrow$ **At $T = 0$ (K), LZ transitions are fully robust under dephasing**.

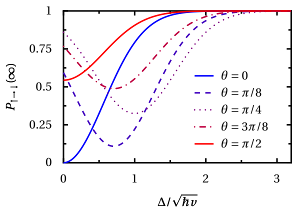
VI. Gauging a quantum heat bath [3]

Central result: Exact transition probability for *arbitrary* bath coupling:

$$P_{\uparrow\to\downarrow}(\infty) = 1 - e^{-\pi W^2/2\hbar v} \quad \text{with} \quad W^2(\Delta) = (\Delta - E_0\sin\theta\cos\theta)^2 + S\sin^2\theta,$$

which involves the reorganization energy $E_0 = \frac{\hbar}{4\pi}\int_0^\infty d\omega\,J(\omega)/\omega$.

Gauging: measure E_0 and $S \Rightarrow$ fix parameters of spectral densities $J(\omega)$.



Idea: vary Δ to find Δ_{min} for which $P_{\uparrow\to\downarrow}(\infty)$ is minimal. Δ_{min} gives E_0 and $P_{\uparrow\to\downarrow}^{min}(\infty)$ gives S. For $J(\omega) = \alpha\omega e^{-\omega/\omega_c}$, $\hbar\omega_c = S/(E_0)$.

Figure 5: Final transition probability $P_{\uparrow\to\downarrow}(\infty)$ as a function of intrinsic interaction Δ, for several values of coupling angle θ. Parameters: $E_0 = 2\sqrt{\hbar v}$ and $S = 0.5\hbar v$. Weak bath coupling gives $E_0^2 \ll S \Rightarrow$ **LZ robust under dephasing**.

VII. Applications

E.g. Circuit cavity QED, nanomagnets, adiabatic quantum computation.

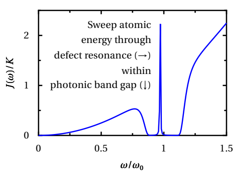


Figure 6: Cavity QED in a photonic crystal. Atom and defect interact.

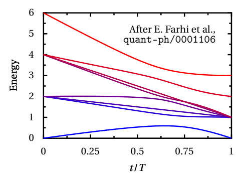


Figure 7: AQC: Ground-state quantum computation. Here a 3-qubit algorithm.

Acknowledgments and references

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